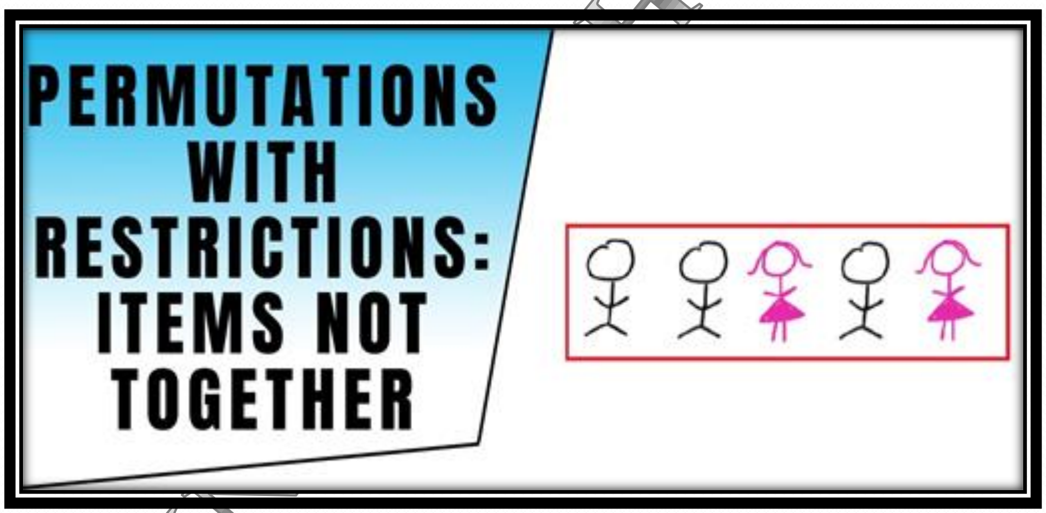
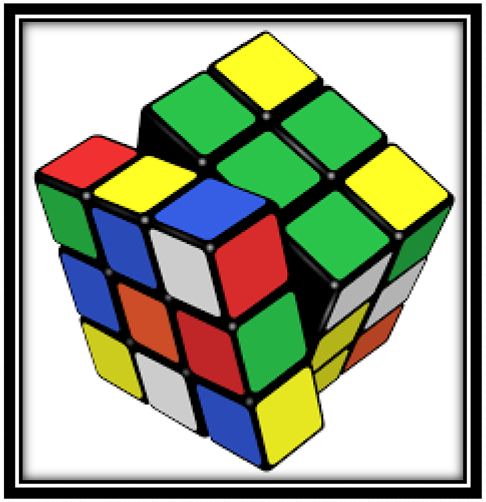
Permutation and Combination

Module-3





Recap

"Fundamental Principle of Counting states that "If an event can occur in **m** different ways , following which another event can occur in **n** different ways, then the total number of occurrence of the events in the given order is **m x n** ."

The notation 'n!' represents the product of first n natural numbers

A Permutation is an arrangement in a definite order of number of objects taken some or all at a time

The number of permutation of n different objects taken r at a time, where $0 < r \leq n$ and the objects do not repeat is $^{n}P_{r}$.

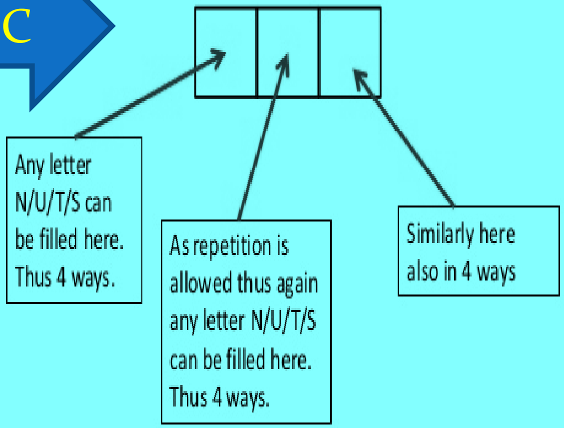
Theorem-2

The number of Permutations of n different objects taken r at a time , when repetition is allowed , is n^r

Example

How many 3 letter words with or without meaning can be formed by word NUTS when repetition is allowed?

Using FPC



i.e. 4 X 4 X 4 = 64 word

How many 3 letter words with or without meaning can be formed by word NUTS when repetition is allowed?

Using Theorem 2

Solution:

Here:

n = 4 (no of letters we can choose from)

r = 3 (no of letters in the required word)

Thus by Theorem 2:

$$n^r = 4^3 = 64$$

Thus 64 words are possible

Find:

The number of 3-letter words which can be formed by the letters of the word NUMBER

No. of permutations of 6 objects taken 3 at a time…

(i) If the repetition is not allowed = 120

$$^6P_3 = \frac{6!}{3!} = 4 \times 5 \times 6 = 120.$$

(ii) If the repetition is allowed=216

$$6^3 = 216$$

PERMUTATIONS CONTINUED

How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if
(i) 4 letters are used at a time,
(ii) all letters are used at a time,
(iii) all letters are used but first letter is a vowel?

i) 6 objects taken 4 at a time= $6P_4 = 360$

ii) 6 objects taken 6 at a time $6! = 720$

iii)

2 x 5! = 240

How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

There are 8 different letters in the word EQUATION.
Therefore, the number of words that can be formed using all the letters of the word EQUATION, using each letter exactly once, is the number of permutations of 8 different objects taken 8 at a time, which is $^8P_8 = 8!$.
Thus, required number of words that can be formed $= 8! = 40320$

Permutation of Non Distinct Objects

Q. Find the number of ways of rearranging the letters of the word ROOT.

Ans. Here there are 4 objects (R,O,O,T),where 2 objects (O , O) are of the same kind and the rest are all different .

$RO_1\,O_2\,T$
$RO_2\,O_1\,T$ → 1

$T\,O_1\,O_2\,R$
$TO_2\,O_1\,R$ → 1

$O_1\,TO_2\,R$
$O_2\,TO_1\,R$ → 1

EVERY 2 ARRANGEMENTS, IS A WORD

4 OBJECTS TAKEN 4 AT A TIME=4!

the required number of permutations = $\frac{4!}{2!} = 3\times 4 = 12$.

SEERR

$SE_1E_2E_3R$
$SE_1E_3E_2R$
$SE_2E_1E_3R$
$SE_2E_3E_1R$
$SE_3E_2E_1R$
$SE_3E_1E_2R$

6

EVERY 6=3! ARRANGEMENTS, IS A WORD

5 OBJECTS TAKEN 5 AT A TIME=5!

the required number of permutations = $\frac{5!}{3!} = 5 \times 4 = 20$.

Theorem 1:

The number of permutations of n objects, where p objects are of the same kind and rest are all different $= \frac{n!}{p!}$

Consider the arrangment of 4 letter word '**ABCD**' when i) no alphabet is repeated (ii) two are same (iii)three are same (iv)all four are same.

ABCD = 24 = 4!

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

A B B D = 12 = $\frac{4!}{\_!}$

ABBD	ABDB	ADBB	
BABD	BADB	BDAB	BDBA
BBAD	BBDA		
DABB	DBAB	DBBA	

ABBB = 4 = $\frac{4!}{\_!}$

ABBB
BABB
BBAB
BBBA

AAAA = 1 = $\frac{4!}{\_!}$

AAAA

If number of alphabets are **5**

all different

ABCDE = 5!
120

2 same

ABBCD = $\frac{5!}{\_!}$
60

3 same

ABBBC = $\frac{5!}{\_!}$
20

4 same

ABBBB = $\frac{5!}{\_!}$
5

all same

BBBBB = $\frac{5!}{\_!}$
1

Permutation of Non Distinct Objects

Theorem 2:
The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is

$$\frac{n!}{p_1!\ p_2!\ \ldots\ p_k!}$$

Example

Find number of permutations of word ALLAHABAD.

Here total number of word **(n) = 9**
Number of repeated A's **(p_1)= 4**
Number of repeated L's **(p_2)= 2**
Rest all letters are different.

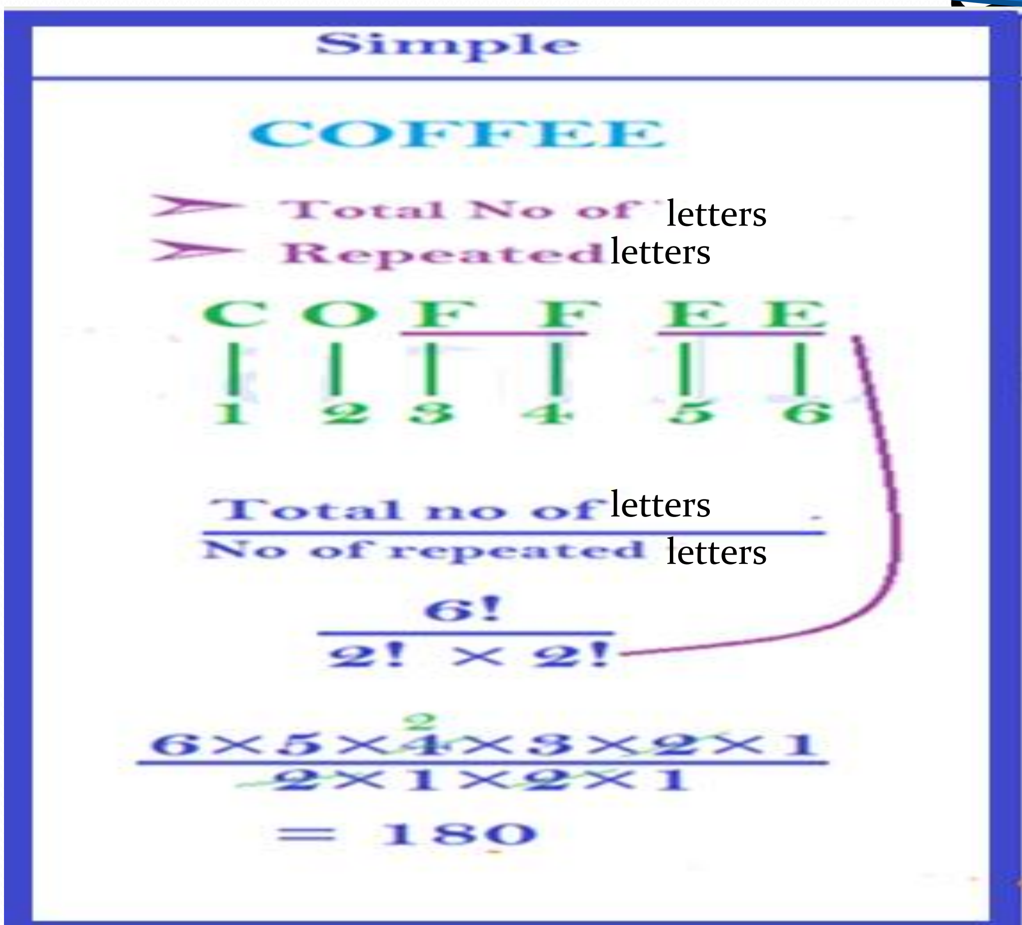
Thus applying theorem , we have:

$$\frac{n!}{p_1!p_2!} = \frac{9!}{4!2!} = 7560\ ways$$

Find the number of arrangements of the letters of the word COFFEE.

How many words can be made from MALAYALAM???

LET'S SOLVE



MALAYALAM

TOTAL =
M comes-
A comes-
L comes-

How many words???

Question 10:
In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Answer 10:

In the given word MISSISSIPPI, I appears 4 times, S appears 4 times, P appears 2 times, and M appears just once.

Total no. of arrangements=34650

$$= \frac{11!}{4!4!2!}$$
$$= \frac{11\times10\times9\times8\times7\times6\times5\times4!}{4!\times4\times3\times2\times1\times2\times1}$$
$$= \frac{11\times10\times9\times8\times7\times6\times5}{4\times3\times2\times1\times2\times1}$$
$$= 34650$$

4 I's do not come together = 34650 – 840 = 33810

No. of arrangements, where 4 I's come together =840

M	S	S	S	S	P	P	I I I I

There are 8 objects in which there are 4 Ss and 2 Ps
therefore no. of ways $\frac{8!}{4!\,2!}$
i.e., 840 ways.

Permutations continued

In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same color are indistinguishable ?

Sol: Total number of discs are 4 + 3 + 2 = 9. Out of 9 discs, 4 are of the first kind (red), 3 are of the second kind (yellow) and 2 are of the third kind (green).
Thus number of permutation is:

$$\frac{9!}{4!3!2!} = 1260$$

- Find the number of the arrangement of all nine letters of word SELECTION in which the two letters E are not next to each other.
- Solutions:

  Total no. of arrangements – No. of arrangements with two E next to each other

$$= \frac{9!}{2} - 8!$$

$$= 141120$$

ASSIGNMENT

1	**How many numbers are there between 100 and 1000 such that 7 is in the unit's place?**
2	How many words beginning and ending with a consonant can be formed by using the letters of the word EQUATION?
3	How many words can be formed out of the letters of the word ORIENTAL so that A and E occupy odd places?
4	Find the number of permutations of the letters of the word HYDERABAD.
5	In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?
	ANSWERS;(i) 90 (2) 4320 (3) 8640 (4) $\frac{9!}{2!2!}$ (5) 14400